

APPLICATION NOTE

Fundamental Radiometry and Photometry

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INTRODUCTION

The responsivity of Kodak image sensors is measured and specified in radiometric units, which have meaning throughout the entire electromagnetic spectrum. Radiometric units are purely physical quantities in contrast to photometric quantities that are based on the response of the human eye. Thus, photometry is the measurement of the ability of electromagnetic radiation to induce a visual sensation in a physically realizable manner, that is, via a defined simulation of human vision, the CIE (Commission Internationale de l'Éclairage) standard observer.

In this note, we provide some of the basic concepts and definitions of radiometry and photometry and show how one can convert from one set of units to the other

BASIC DEFINITIONS AND CONCEPTS

In the following discussion, only basic concepts and definitions of radiometry and photometry are discussed. More detailed discussions can be found in the references listed in the bibliography at the end of this note.

A source emits a radiant flux (Φ_e) that is proportional to the area enclosed by its spectral distribution curve:

$$j_e = \int_0^{\infty} j_{e,\lambda} d\lambda \quad (\text{watts}) \quad \text{Eq. 1}$$

Radiance is the radiant flux per unit area and unit solid angle arriving at or leaving from a surface from a given point:

$$L_e = \frac{dj_e}{dA \cos \theta d\Omega} \quad (\text{w m}^{-2} \text{sr}^{-1}) \quad \text{Eq. 2}$$

Now, to determine the capacity of the radiant flux to create the sensation of light, the $j_{e,\lambda}$ curve must be transformed to a luminous flux.

This is accomplished by multiplying $j_{e,\lambda}$ by the photopic relative luminous efficiency function $V(\lambda)$.

$$j_v = \int_{380}^{780} j_{e,\lambda} V(\lambda) d\lambda \quad (\text{watts}) \quad \text{Eq. 3}$$

$V(\lambda)$ is the ratio of the radiant flux at wavelength λ_m to that at wavelength λ , when the two fluxes produce the same photopic luminous sensation under specified photometric conditions. λ_m is chosen so that the maximum value of this ratio is unity. In essence, $V(\lambda)$ is the spectral response of the human eye normalized so that it has the value 1 at 555nm, the peak of the human eye's response. Table 1 lists the values of $V(\lambda)$ and Figure 2 shows the response graphically.

Since $V(\lambda)$ is dimensionless, Φ_v remains in units of watts, although the units might be better thought of as lightwatts as we are now in the visual domain.

To convert the luminous flux from watts (lightwatts) to the photometric equivalent, lumens, multiply Eq. 2 by the maximum luminous efficacy factor, K_m :

$$j_v = K_m \int_{380}^{780} j_{e,\lambda} V(\lambda) d\lambda \quad (\text{lumens}) \quad \text{Eq. 4}$$

For photopic vision the maximum luminous efficacy of radiant flux is 683 lumens watt⁻¹.

Luminance is the photometric unit corresponding to radiance and is defined in the same way:

$$L_v = \frac{dj_v}{dA \cos \theta d\Omega} \quad \text{Eq. 5}$$

The luminous efficacy of the total radiant flux is represented by the symbol K and is the product of the luminous efficiency V and the maximum luminous efficacy factor K_m

$$K = V \cdot K_m = \frac{j_v}{j_e} \quad \text{Eq. 6}$$



Table 1. Photopic relative luminous efficiency function V(λ) and luminous efficacy of the standard observer.

λ (nm)	V(λ)	K(λ)	λ (nm)	V(λ)	K(λ)
380	0	0	580	0.87	594.21
390	0.0001	0.0683	590	0.757	517.031
400	0.0004	0.2732	600	0.631	430.973
410	0.0012	0.8196	610	0.503	343.549
420	0.004	2.732	620	0.381	260.223
430	0.0116	7.9228	630	0.265	180.995
440	0.023	15.709	640	0.175	119.525
450	0.038	25.954	650	0.107	3.081
460	0.06	40.98	660	0.061	41.663
470	0.091	62.153	670	0.032	21.856
480	0.139	94.937	680	0.017	11.611
490	0.208	142.064	690	0.0082	5.6006
500	0.323	220.609	700	0.0041	2.8003
510	0.503	343.549	710	0.0021	1.4343
520	0.71	484.93	720	0.001	0.683
530	0.862	588.746	730	0.0005	0.3415
540	0.954	651.582	740	0.0003	0.2049
550	0.995	679.585	750	0.0001	0.0683
560	0.995	679.585	760	0.0001	0.0683
570	0.952	650.216	770	0	0

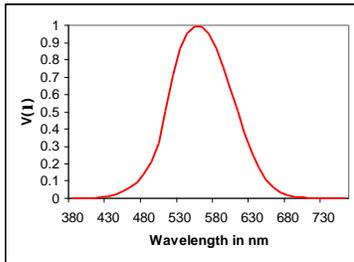


Figure 1. Photopic response of the human eye (CIE standard observer).

When luminous flux is expressed in lumens and not watts, then $V(\lambda)$ becomes spectral luminous efficacy of radiant flux $K(\lambda)$. It is expressed in lumens·watt⁻¹:

$$K(\lambda) = V(\lambda) \cdot K_m \tag{Eq. 7}$$

From the foregoing, then, knowing the radiant flux incident on the detector, it is possible to convert to a luminous flux, which is the desired quantity.

A table of radiometric and photometric terms and units showing their relationships is provided in Appendix I.

CONVERSION FROM RADIOMETRIC TO PHOTOMETRIC UNITS

Conversion of the radiometric responsivity (R_e) of a sensor to photometric responsivity (R_v) first requires knowledge of the light source. If the source is specified as having a certain color temperature, and we assume that its exitance is the same as a perfect blackbody radiator, then its spectral radiance, in W·cm⁻² nm⁻¹ sr⁻¹ is given by Planck's Law

$$L_{\epsilon\lambda} = \frac{3.741 \times 10^{19}}{\lambda^5 \left[e^{(14388000/\lambda T)} - 1 \right]} \tag{Eq. 8}$$

where λ is in nm and T is in Kelvin.

Artificial sources in general do not have the same spectral distribution as a perfect blackbody but for our purposes, we shall consider them equal. Figure 2 depicts the spectral radiance of several blackbody radiators.

Sensor responsivity is derived from the relationship between the number and energy of photons arriving at the sensor, the ability of the sensor to convert them into electrons and ultimately into an output voltage. For an incident beam with radiant flux Φ_λ and photons with energy $E = h\nu$, the number of photons arriving at the detector is $\Phi_\lambda/h\nu$. The detector converts photons to electrons with quantum efficiency ξ so the number of electron produced per unit time is $N_{e^-} = \xi \Phi_\lambda/h\nu$.



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If each electron contributes to the output signal, then the voltage is $N_e \cdot O$, where O is the product of charge-to-voltage conversion and the internal sensor gain. It is measured in units of volts/e⁻. The voltage then is $V_\lambda = (\xi O / h\nu) \phi_\lambda$. Spectral responsivity is

$$R(\lambda) = V_\lambda / \phi_\lambda = \xi O / h\nu \quad \text{Eq. 9}$$

Integrating over the wavelength range that the sensor is sensitive yields the total responsivity

$$R = \frac{\int V_i \, dI}{\int j_i \, dI} \quad \text{Eq. 10}$$

The output of the sensor, ignoring integration time, becomes

$$V_{\text{out}} = L_e \cdot R \quad \text{Eq. 11}$$

The same analysis applies for responsivity in terms of luminous flux. Consequently, we can arrive at an expression for the photometric responsivity, R_v , as a function of the radiometric responsivity, R_e :

$$R_v = \frac{\int L_{e\lambda} R_{e\lambda} \, dI}{K_m \int L_{e\lambda} V(\lambda) \, dI} \quad \text{Eq. 12}$$

This is best shown by example.

Example 1.

A Kodak Digital Science KLI-8013 trilinear sensor with the standard CFA is used in an imaging application that uses a 3200K halogen light source and a BG-38 IR cutoff filter glass 2 mm thick. The blue channel radiometric responsivity is listed in column 2 of Table 2 in units of $V \cdot \mu J^{-1} \cdot \text{cm}^{-2}$. What is the photometric responsivity of the blue channel in $V \cdot (\text{lux} \cdot \text{sec})^{-1}$?

For simplicity, and since this is a relative comparison, we ignore integration time and the effects caused by any optical elements in the imaging system.

First we must calculate the spectral radiance of the lamp or obtain the information from actual measurements (or manufacturer's data). Calculated radiance (Eq. 8) for a 3200K source is shown in column 3 of Table 2.

The actual radiance incident on the sensor is shaped by the effect of the BG-38 filter glass

$$L_{e\lambda, \text{net}} = L_{e\lambda} \cdot \tau_{\text{BG-38}}$$

where, $\tau_{\text{BG-38}}$ is the transmittance through the filter glass (column 4).

For practical purposes, we can replace the integration over the visible spectrum of infinitely small strips of wavelength width $d\lambda$ with the summation of strips of finite width $\Delta\lambda$ (10nm in this case). Thus, the rest of the calculation is straightforward and is detailed in columns 6 and 7.

The result is that the blue channel photometric response of this sensor and CFA, illuminated by a 3200K source is

$$R_v = \frac{100 \times 2.78 \times 10^4}{4.12 \times 10^6} = .675 \text{ V lux}^{-1} \text{ sec}^{-1}$$

(Note: The additional factor of 100 is a result of converting μJ to watts and m^2 to cm^2 .)

Similarly, the green channel response is $1.4 \text{ V lux}^{-1} \text{ sec}^{-1}$, and the red channel response is $3.45 \text{ V lux}^{-1} \text{ sec}^{-1}$.

This exercise shows that without full knowledge of all system parameters – optical system throughput, light source spectral radiance, filter glass transmission, and sensor responsivity – conversion from radiometric to photometric units is not possible.



Table 2. Values of spectral radiance and filter glass transmission used to calculate luminous responsivity from radiometric responsivity.

Wavelength	Response	Spectral Radiance		Net Radiance	$L_{el} * R_l$	$L_{el} * V(\lambda)$
	$V \text{ mJ}^{-1} \text{ cm}^{-2}$	3200K Source $W \text{ cm}^{-2} \text{ nm}^{-1} \text{ sr}^{-1}$	BG-38 Transmission			
360	0.03	7.42E+00	0.798	5.92E+00	1.75E-01	0.00E+00
370	0.09	9.06E+00	0.834	7.56E+00	6.85E-01	0.00E+00
380	0.39	1.09E+01	0.861	9.40E+00	3.64E+00	0.00E+00
390	1.17	1.30E+01	0.87	1.13E+01	1.33E+01	1.13E-03
400	2.48	1.53E+01	0.877	1.34E+01	3.32E+01	5.36E-03
410	4.26	1.78E+01	0.882	1.57E+01	6.67E+01	1.88E-02
420	5.78	2.04E+01	0.886	1.81E+01	1.05E+02	7.24E-02
430	7.10	2.33E+01	0.891	2.08E+01	1.47E+02	2.41E-01
440	8.29	2.63E+01	0.893	2.35E+01	1.95E+02	5.41E-01
450	9.18	2.95E+01	0.897	2.65E+01	2.43E+02	1.01E+00
460	9.41	3.29E+01	0.899	2.96E+01	2.78E+02	1.77E+00

700	0.11	1.15E+02	0.224	2.58E+01	2.84E+00	1.06E-01
710	0.37	1.18E+02	0.187	2.20E+01	8.16E+00	4.61E-02
720	1.23	1.20E+02	0.155	1.85E+01	2.27E+01	1.85E-02
730	3.19	1.22E+02	0.128	1.56E+01	4.97E+01	7.79E-03
740	6.57	1.24E+02	0.106	1.31E+01	8.61E+01	3.93E-03
750	10.22	1.25E+02	0.087	1.09E+01	1.11E+02	1.09E-03
760	14.22	1.27E+02	0.072	9.14E+00	1.30E+02	9.14E-04

$S_{L_{el}} R_{el} \Delta\lambda$ 683 $S_{L_{el}} V(\lambda) \Delta\lambda$
2.78E+04 4.12E+06



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Blackbody Radiance Curves

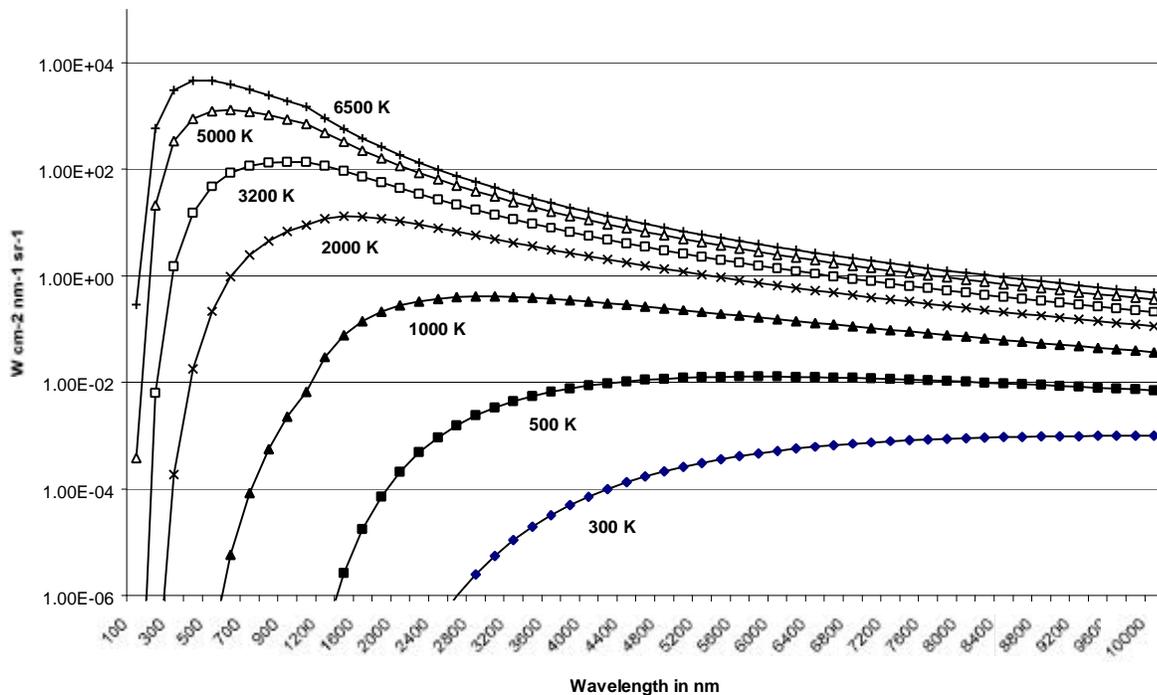


Figure 2. Blackbody radiation curves for various sourcecolor temperatures, calculated from Planck’s Law.



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APPENDIX I

Table 3. Fundamental Radiometric and Photometric Quantities

Fundamental Radiometric Quantities			Fundamental Photometric Quantities		
<i>Quantity</i>	<i>Symbol</i>	<i>Units</i>	<i>Quantity</i>	<i>Symbol</i>	<i>Units</i>
Radiant energy	Q_e	J (joule)	Luminous energy	Q_v	lm s
Radiant energy density	w_e	J m ⁻³	Luminous energy density	W_v	lm s m ⁻³
Radiant power or flux	ϕ_e	W (watt)	Luminous flux	ϕ_v	lm
Radiant exitance	M_e	W m ⁻²	Luminous exitance	M_v	lm m ⁻²
Irradiance	E_e	W m ⁻²	Illuminance	E_v	lm m ⁻²
Radiant Intensity	I_e	W sr ⁻¹	Luminous intensity	I_v	cd = lm sr ⁻¹
Radiance	L_e	W m ⁻² sr ⁻¹	Luminance	L_v	cd m ⁻²
Emissivity	ε	-	Luminous efficacy	K	lm W ⁻¹
			Photopic luminous efficiency	$V(\lambda)$	-
			Maximum spectral luminous efficacy	K_m	lm W ⁻¹

(cd is candela, lm is lumen, sr is steradian)



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